

From Music to Mathematics: Exploring the Connections

Gareth E. Roberts

Recommended Music to Accompany Chapter 1: Rhythm

1. John Philip Sousa, *The Stars and Stripes Forever*, 1896. Track 58 on a CD compiled by Wynton Marsalis titled Music Examples to Accompany Marsalis on Music, Sony Classical. This piece is in cut time (C with a vertical slash through the middle), which is equivalent to a $\frac{2}{2}$ meter. Incidentally, this is the official march of the USA.
2. Pyotr Ilyich Tchaikovsky, *The Nutcracker Suite: Russian Dance*, Op. 71A, 1892. Track 6 from the Fantasia Motion Picture Soundtrack, the Walt Disney Company. This lively dance in $\frac{2}{4}$ time accelerates to a climactic finish. Notice Tchaikovsky's use of the tambourine.
3. Elvis Crespo, *Suavemente*, Merengue. Track 1 off of a hit CD of the same name on the Sony label, 1998. The album reached the top of the Latin billboard charts in the US in 1998. Merengue is a popular style of music and dance originating in the Dominican Republic that features a fast rhythmic two-count in $\frac{2}{4}$ meter.
4. Pyotr Ilyich Tchaikovsky, *The Nutcracker Suite: Waltz of the Flowers*, Op. 71A, 1892. Track 7 from the Fantasia Motion Picture Soundtrack, the Walt Disney Company. This waltz, like most, is in $\frac{3}{4}$ time. To count along, say "one-two-three" along with the "oom-pah-pah" in the lower strings. Or, dance to it, following the "down-up-up" pattern.
5. Billy Joel, *The Piano Man*, 1973. Track 1 on Disc 1 of his Greatest Hits album, Blackwood Music Inc. The $\frac{3}{4}$ meter rarely finds its way into popular rock music as it does here with this classic from one of the founders of piano rock. This meter is more common in the country/western genre. Billy Joel studied classical music at an early age and as a result, much of his music shows a deeper melodic and rhythmic sophistication than most pop music.
6. Franz Gruber, *Silent Night* (lyrics by Josef Mohr), 1820. Track 3 on a Windham Hill CD titled Windham Hill: The Night before Christmas, performed by Barbara Highbie, Sony BMG Music Entertainment. This famous Christmas carol is an excellent example of $\frac{6}{8}$ meter. Note the difference with the two previous pieces in $\frac{3}{4}$ time. Although it is possible to count along with the piece in 3, it is most definitely not a waltz. Music in $\frac{6}{8}$ time often has a swaying, singsongy feel to it and the typical bar is usually subdivided in two rather than six. The music for *Silent Night* is shown in Figure 1.5. The English folk song *Greensleeves* is another popular example of this compound meter.
7. R.E.M., *Everybody Hurts*, 1992. Track 4 off the band's 8th album Automatic for the People, on the Warner Bros. label. This somber ballad is also in $\frac{6}{8}$ time, a rarity in rock music. The song was rearranged by Simon Cowell to raise relief funds for Haiti after the devastating earthquake that occurred there on January 12, 2010. The song was listed by the PRS Music society as the #1 song to make "real men" cry, and was once voted "the saddest song of all time" in a British poll.

8. Leonard Bernstein, *America* (lyrics by Stephen Sondheim), 1957. This famous song, from the musical *West Side Story*, repeatedly alternates between $\frac{6}{8}$ and $\frac{3}{4}$ time (see Figure 1.6 in the textbook). Each odd-numbered measure is in $\frac{6}{8}$ and features two groupings of three eighth notes, while the even-numbered measures are in $\frac{3}{4}$ and contain three quarter notes (rhythmically equivalent to three groups of two eighth notes). Even though the time signature changes in consecutive measures, the eighth-note pulse is constant throughout. The odd measures feel in two, while the even measures are in three. Bernstein's musical depiction of the commutative property ($2 \cdot 3 = 3 \cdot 2$) creates a memorable rhythmic pattern with a distinctive Latin flavor, a nice touch of irony given that the lyrics glorify the benefits of living in "America."
9. Pyotr Ilyich Tchaikovsky, *Allegro con grazia*, 2nd movement of the Symphony, No. 6, "Pathétique", Op. 74, 1893. Track 2 from a Chicago Symphony recording, conducted by Claudio Abbado, CBS Records. The entire second movement of this symphony is in $\frac{5}{4}$ time, a very unusual meter for its day. The five beats of each measure can be broken down into $2 + 3$ giving the music its dance-like character (see Figure 1.7 in the textbook). Try counting along directly from the start of the movement by repeating "one-two one-two-three" over and over again.
10. Paul Desmond, *Take Five*, 1959. Track 3 on the first side of the best-selling jazz album *Time Out* by The Dave Brubeck Quartet on the Columbia label. As the name indicates, this catchy tune is also written in $\frac{5}{4}$ time and is offered here as a contrast to the classical piece in 5 by Tchaikovsky. Unlike that piece, here the division of each measure is into $3 + 2$ (see Figure 1.8 in the textbook). Try counting along to feel the subdivision. Brubeck was inspired to produce jazz with alternative time signatures after hearing some Turkish music in $\frac{9}{8}$. The tune *Blue Rondo à la Turk*, on the same album, pays homage to its Turkish roots in name and through its $\frac{9}{8}$ meter.
11. Peter Gabriel, *Solsbury Hill*, 1977. Track 2 on the first side of Gabriel's first studio album titled *Peter Gabriel*, on the Atco label. This reflective song was written by Gabriel based on a spiritual experience he had atop Solsbury Hill in Somerset England, just after leaving the band Genesis. Although there are seven beats to a measure, the music combines so well with the underlying meter that it is hard to notice the unique time signature without counting along. Each measure is subdivided into a $3 + 4$ pattern. There is also a nice *hemiola* (a 3 against 2 pattern) on the words "boom boom boom" in the chorus section. In this case, the hemiola is a quarter-note triplet, where three notes are equally spaced over two quarter-note beats. Pink Floyd's *Money* (1973), notable for its unique opening featuring money-related sound effects, is another good example of a rock tune in $\frac{7}{4}$ time.
12. Cuarteto Oriente, *Mueve La Cintura Mulata*, Cuban Son music. Track 8 on the Putumayo CD *Afro-Latin*, Putumayo World Music, 1998. This piece features the common Afro-Cuban 3-2 son clave rhythmic pattern (see the top of Figure 1.9 in the textbook). The pattern derives from West African music and nearly all Afro-Cuban music (including salsa) centers around this famous clave rhythm. Because of its significance, the 3-2 rhythmic pattern is an example of a *timeline*, a recurring rhythmic motif that is central to the piece and to an entire genre of music. Try and tap out the 3-2 clave rhythm produced by the claves, two small wooden sticks that are struck together to produce a sharp, percussive sound.

13. Viejá Trova Santiaguera, *Me Dieron La Clave*, Cuban Guaracha. Track 11 on the Putumayo CD Afro-Latin, Putumayo World Music, 1998. In contrast to the previous piece, the rhythmic underpinning here is the 2-3 clave rhythm (see the bottom of Figure 1.9 in the textbook). Can you hear the difference?
14. Coldplay, *Clocks*, 2002. Track 5 off the band's second album A Rush of Blood to the Head, released in the US by Capitol Records. This song is featured here due to its repeated use of the same rhythmic pattern in the piano, a technique called an *ostinato*. By accenting the same pulses in each measure, the repeated eighth-note pattern subdivides each measure into $3 + 3 + 2$. This $3 + 3 + 2$ subdivision of the measure is a syncopated pattern that is particularly noteworthy. In fact, it is the exact same subdivision used in the first measure of the 3-2 son clave pattern (upper-right portion of Figure 1.9). This particular subdivision is common in music from all over the world, including Central Africa, Cuba, the Middle East, and the United States. It is featured prominently on the banjo for bluegrass music, and was popular in early Rock and Roll (e.g., the bass line in Elvis Presley's version of *Hound Dog* (1956)). It is also the basis for the famous *Charleston* dance rhythm. In *Clocks*, the subdivision underlies the entire song, a creative way to enliven a simple $\frac{4}{4}$ meter, the actual time signature for the song.
15. *Raga for Tabla* (Indian classical music). Track 2 from a CD entitled The Best of India, Madacy Entertainment Group, Inc. 2001. (Unfortunately, no information is given concerning the names of the performers.) The tabla is a common Indian percussion instrument central to Indian classical music, consisting of two drums, one for each hand (see Figure 1.11 in the textbook). The two hand drums are different sizes and timbres. Often, a tabla player is asked to simultaneously subdivide a measure into a different numbers of beats for each hand, (say 7 versus 11), a task that requires years of training. As discussed in Section 1.3 "Polyrhythmic Music", the least common multiple is the underlying mathematical principle at work. Notice the variety and complexity of the rhythms in this piece.
16. *Improvisations for Tabla and Vamsa* (Indian classical music). Track 6 from the previous CD featuring a creative collaboration between tabla and the bamboo flute *vamsa*. The young mathematician/musician Manjul Bhargava at Princeton University (who just won the Fields medal – the highest honor for a mathematician), stressed the importance of mathematical thinking in performing and understanding Indian classical music. The French composer Olivier Messiaen was aware of this importance and utilized many Indian rhythms in his music. Click the button on the LINKS page of the website to listen to Bhargava play the tabla and hear an interview with him on National Public Radio discussing some of the connections between math and music.
17. Babatunde Olatunji, *Ajaja*, 1988. Track 1 from a CD entitled Drums of Passion: The Invocation, Rykodisc, 360° Productions. This polyrhythmic African music features an array of performers on various percussion instruments such as the Djembe drum (the drum I played in class), the Talking drum, the Hoop drum, the Junjun drum and cowbell. Try and distinguish each individual part, noting both its independent motif as well as how it contributes to the entire ensemble. Note that the title of the piece is a palindrome, a written symmetry. We will discuss mathematical and musical symmetry in great detail next semester.

18. Phish, *First Tube*, 2000. Track 12 from their CD *Farmhouse*, Elektra Records. The fusion rock group Phish draws much of their musical inspiration from jazz, blues, and bluegrass. They are known for their live improvisations and free-flowing form. This is an interesting polyrhythmic song that juxtaposes different rhythms between the drums and guitar. It was nominated for a Grammy in the category of Best Rock Instrumental Performance.

19. Igor Stravinsky, *Le Sacre Du Printemps* (The Rite of Spring), 1913. Track 5 from a recording by the Cleveland Orchestra conducted by Pierre Boulez (Deutsche Grammophon, 1992). This is the first of an amazing two-part piece “Scenes of Pagan Russia.” One of the great orchestral masterpieces of the 20th century, this work features dissonance, polytonality, and polyrhythms. The work premiered as a ballet in Paris and the jarring rhythms accompanied with the exotic, sexual dance led to a riot in the theater! Notice the way Stravinsky uses a variety of orchestral sounds to create passion and fury in his music, all the while keeping it rhythmically vibrant through ostinato (repetition of the same note,) multiple meters (at one point he changes from $\frac{5}{4}$ to $\frac{7}{4}$ to $\frac{6}{4}$ in consecutive measures) and polyrhythmic creativity. Approximately 3.5 minutes into the recording (3:30) begins the eighth-note repetition in the strings with the unexpected, striking accents (see Figure 1.10 in the text). Later on, around 13:30, Stravinsky begins to paint his dramatic polyrhythmic collage, almost as if two competing orchestras are trying to establish their own rhythmic will over each other. This is a marvelous piece of music to see performed live by a huge orchestra. The work was featured prominently in the opening ceremonies of the 2014 Winter Olympics in Sochi, Russia.

20. The National, *Fake Empire*, 2008. Track 1 off the band’s 4th studio album titled *Boxer*, on the Beggar’s Banquet label. This is a fabulous example of a polyrhythmic rock song. Written by Matthew Berninger and Bryce Dessner, the song features a four-against-three polyrhythm throughout the entire piece, perhaps the first rock tune to ever accomplish such a feat. The main piano part is shown in Figure 1.18 of the textbook. Both the right-hand of the piano and the guitar play in four, while the left-hand of the piano, vocals and drums play in three! Dessner, a classically trained guitarist who is also a member of the avant-garde ensemble Bang on a Can, strives to expose his rock audiences to some of the more interesting forms from the modern classical world of music. In addition to the polyrhythm, Australian composer Padma Newsome, who was asked to write horn parts for the end of the song, provides a minimalist feel in the style of Steve Reich (see Section 8.2 of the text) to close the piece. Although Dessner describes the piano part and its definitive polyrhythm as “*Chopsticks* simple,”¹ ultimately, the polyrhythm is what distinguishes this remarkable song.

¹Derk Richardson, *The National’s brand of intelligent art-rock quietly hooks listeners in*, San Francisco Chronicle (June 21, 2007)