From Music to Mathematics: Exploring the Connections

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Recommended Music to Accompany Chapters 6, 7, and 8

These selections showcase some modern "mathematical music" from the twentieth century, as well as a few examples of change ringing from England. Featured here are a twelve-tone piano piece by Schoenberg, a composition based entirely on an 8×8 magic square by Davies, an orchestral work using "stochastic music" by Xenakis, and two creative rhythmic works by minimalist composer Steve Reich.

As discussed in Section 2.4.5 of the text, a rather radical shift in Western music occurs at the start of the twentieth century. Many composers discarded tonality altogether, seeking more freedom and flexibility in utilizing all the notes of the chromatic scale, rather than favoring one over the others. Arnold Schoenberg wrote, "Tonality is not an eternal law of music, but simply a means toward the achievement of musical form." Despite this assessment, it is interesting to note that Schoenberg still utilized the same musical ideas as Bach (inversion, retrograde, transposition, canons) to build his music out of a given tone row. Davies used the magic square of Mercury to serve as an architectural blueprint for his innovative composition, intended to depict the remarkable properties of light. Xenakis took the atonality concept even further, composing "sound blocks" based on some very precise mathematical calculations involving probability theory. Finally, Reich shows a knack for making slight adjustments rhythmically (shifting or altering accents) to generate whole compositions out of short and simple musical statements.

1. *Little Bob Maximus*, St. Mary Redcliffe, Bristol, England. A recording of the first 176 changes from Little Bob Maximus, this is Track 1 from the SayDisc Records CD Church Bells of England (1989). The church of St. Mary Redcliffe was described by Queen Elizabeth the First as "the faerest and goodliest parish church in all my realm." Many of the bells were cast in 1903 with numbers 8 and 10 dating to 1763 and number 11 all the way back to 1622. A "Flat" Sixth bell was added in 1951 while a "Sharp" Treble rounded out the collection in 1970. Notice that the excerpt starts on rounds (moving in pitch from highest to lowest) but after about a minute, enough permutations have occurred that the pitch moves in completely the opposite direction (from lowest to highest).

2. *Grandshire Doubles*, St. Bartholomew the Great, Smithfield, England. This is track 11 off the previous CD. St. Barthlolomew's church in London was founded in 1123 as an Augustinian Priory. The five bells heard here were cast by Thomas Bullisdon of Aldgate in about 1510 and are the only complete medieval set of more than four from one creator in England. This recording is of the usual band of Sunday bell ringers at the church. Due to the fact that there are only five bells, it is easier to follow the different changes as they progress through the piece.

3. Arnold Schoenberg, *Suite für Klavier*, (Suite for Piano; 1923), op. 25. Tracks 15–21 on a Deutsche Grammophon CD titled Schoenberg: The Piano Music performed by Maurizio Pollini. This is Schoenberg's first piece to use the 12-tone method throughout. The entire work is based on the same primary tone row (see Section 7.2 of the text). The piece is atonal in the sense that there is no central key or tonic around which the music is based. Schoenberg uses only the six tone rows P_0, P_6, I_0, I_6, R_6, and RI_6. The number six is significant in that six half-steps equals half the octave (the tritone interval) and the tone row begins on an E and ends a tritone away on a B♭ .

Each of the transformations above (transposition, inversion, retrograde and retrograde-inversion) all consequently begin and end on either E or B♭ , making the music flow easier from one row to another. These tone rows, as well as some excerpts from different parts of the *Suite*, can be found in Figures 7.2–7.6 of the textbook.

4. Peter Maxwell Davies, *A Mirror of Whitening Light*, op. 75, 1977. This live recording of the Manson Ensemble, conducted by Diego Masson, was downloaded from the composer's website `www.maxopus.com` (Sir Davies was widely known as "Max"). The title of the piece refers to the alchemical process of purification or "whitening" of a base metal into gold. In the opening lecture before the first broadcast performance, Davies describes how the light outside his window workplace in Orkney gave him inspiration for the piece. The composition relies on an 8×8 magic square (the magic square of Mercury; see Table 8.4 in the text) to create the entire set of notes and durations for the piece. Although he claimed in his lecture to have used the magic square of the sun (a 6×6 square), presumably wanting to embody light in some manner, it is clear that Davies actually used the magic square of Mercury instead.

   Each number in the square corresponds to a particular note from an 8-note plainchant *Veni sancte spiritus* or its transposed version. The original was transposed to start on each of the eight notes of the chant, thus creating a total of eight 8-note phrases (see Figure 8.6). These 64 notes were then mapped onto the 8×8 magic square of Mercury (see Table 8.5). Davies uses patterns from his musical magic square to assign notes and durations to the different instruments. He writes, "I see these patterns, in the first place, possibly as dance patterns; and one gets to know them by heart. One doesn't in fact deal with numbers at all. One deals rather as somebody who is dealing with *bell-changes, with actual patterns with changes*" (emphasis added). Concerning the seemingly rigid structure imposed by the magic square, Davies states, "I firmly believe that the more one controls the flow of one's wildest inspiration, the wilder it sounds. And so when I really wanted to be wild towards the climax of this work, I imposed very rigid rhythmic and tonal controls derived from the plainsong, and from that magic square; and the result is really quite extraordinary I find, even now."

5. Iannis Xenakis, *Pithoprakta* (Actions through Probabilities), 1955–1956. Track 4 from a Le Chant Du Monde CD titled Xenakis: Eonta, Metastasis, Pithoprakta featuring the Orchestre National de l'O.R.T.F. under the direction of Maurice Le Roux. This piece was dedicated to Hermann Scherchen, who conducted its premiere in March 1957 in Munich. This highly mathematical work is written for 46 strings (remarkably all playing different parts), 2 trombones, a xylophone, and a wood block. Xenakis uses probability theory to determine what notes should follow each other and what lengths they should be played, a feature of "Stochastic Music," invented by the composer. The piece explores the conflict between continuity and discontinuity by juxtaposing continuous sounds (glissandi in the strings and trombones) with discontinuous ones (pizzicati plucking in the strings, tapping the wood of the strings with the opposite side of the bow, and the sharp, piercing sounds of the wood block.) Xenakis determines the "speed" of a given glissando (the slope found as the ratio of pitch to duration; see Figure 8.18 in the text) by using a uniform distribution (sometimes called a Gaussian normal distribution.) This has the mathematical effect of distributing the speeds equally among all players so that all pitches freely occur along a continuous frequency spectrum (see Figure 8.19 for an example). In this way, Xenakis extends Schoenberg's main goal in 12-tone music of not favoring any one pitch (in this case frequency) over another. Given Xenakis' experiences with the Greek Resistance and WWII, it is fairly obvious that this work is the composer's musical reflection upon the most tragic of human conflicts—war.

6. Steve Reich, *Clapping Music*, 1972. Track 2 from Disc 3 of a collection of the composer's works titled Steve Reich: Works 1965–1995 produced by Nonesuch Records. This performance, featuring two players clapping, includes the composer himself and Russell Hartenberger. This simple, yet clever work, consists of 13 measures, each with 6 beats and each repeated 12 times for a total of 156 measures or 936 beats. One performer repeats the exact same rhythmic pattern (see Figure 8.11 of the text) repeatedly throughout the work. The other performer begins in unison with the first player, but then gradually shifts the rhythmic pattern to the left (a left cyclic shift) by an eighth note at each new bar. Since there are 6 beats or 12 eighth notes, this will produce a new rhythmic pattern in each measure until returning in unison at the 13th measure. Since the original rhythmic pattern is asymmetric, the second performer has a different pattern at each stage of the eighth-note shift. Surprisingly, this simple shift generates entirely new musical ideas throughout the piece. At times the music consists of repeated eighth notes (when the original pattern and its shifted version are completely out of phase), while at other times, the music has a dance-like feel due to simultaneous rests that occur when the original and shifted pattern are more closely aligned.

   The underlying mathematical structure in the composition is $\mathbb{Z}_{12}$, the cyclic group of order 12, the same group that equates notes in different octaves and that arises in 12-tone music. As is typical with his music, Reich gives very precise instructions to the performers on tempo, duration of the piece (approximately 5 minutes), method of clapping, placement of downbeats, performance style, and even electronic amplification. Ironically, Reich does not follow his own instructions, as the first and last measures (the two unison bars) on this recording are repeated only 6 times, not 12. Try and follow the music as each new left shift occurs.

7. Steve Reich, *Six Marimbas*, 1986. Track 3 from Disc 3 of a collection of the composer's works titled Steve Reich: Works 1965–1995 produced by Nonesuch Records. This performance was recorded by the group *Steve Reich and Musicians* with members of *The Manhattan Marimba Quartet*. The piece is a rescoring for marimbas of Reich's 1973 composition *Six Pianos* and is a more sophisticated example of the rhythmic phase shifting used in *Clapping Music*. By making subtle and gradual rhythmic and melodic changes in the music, a technique often called *minimalist*, the composer creates an almost psychedelic listening experience. The piece opens with four marimbas playing an identical eight-beat rhythmic pattern on different notes in the key of D♭ major. The other two marimbas then enter together, at first playing only one beat per measure, then two, then three and so on until they are playing the same rhythmic pattern. However, the players shift the pattern at the outset, moving it forward two beats (a right shift). For example, they open by playing the notes from beat 5 of the primary pattern on beat 7, then playing the notes from beat 1 on beat 3, etc. Once this rhythmic canon is complete, other players begin to accentuate interesting melodic features of the original quartet. For example, in measure 18 (roughly 1:43 into the recording), player 6 begins a simple ascending run up and over the top of the scale. This stands out until player 5 begins to emphasize a different motif in measure 22. The dynamics of the piece fluctuate as different players momentarily enter or leave the ensemble. Harmonically, the piece is structured into three sections with the first in D♭ major, the second in E♭ dorian and the third in B♭ natural minor. It is instructive to watch a live performance of this piece. A performance at the University of Kentucky can be seen here: `https://www.youtube.com/watch?v=wUE7IA__NZM`