Contents

Preface xi
Acknowledgments xv
Introduction xvii

1 Rhythm 1
1.1 Musical Notation and a Geometric Property 1
1.1.1 Duration: Geometric sequences 2
1.1.2 Dots: Geometric series 4
1.2 Time Signatures 9
1.2.1 Musical examples 10
1.2.2 Rhythmic repetition 13
1.3 Polyrhythmic Music 17
1.3.1 The least common multiple 19
1.3.2 Musical examples 22
1.4 A Connection with Indian Classical Music 28
References for Chapter 1 31

2 Introduction to Music Theory 33
2.1 Musical Notation 34
2.1.1 The common clefs 34
2.1.2 The piano keyboard 37
2.2 Scales 41
2.2.1 Chromatic scale 42
2.2.2 Whole-tone scale 44
2.2.3 Major scales 45
2.2.4 Minor scales 49
2.2.5 Why are there 12 major scales? 50
2.3 Intervals and Chords 55
2.3.1 Major and perfect intervals 56
2.3.2 Minor intervals and the tritone 57
2.3.3 Chords 59
2.4 Tonality, Key Signatures, and the Circle of Fifths 64
2.4.1 The critical tonic-dominant relationship 65
2.4.2 Key signatures 67
2.4.3 The circle of fifths 69
2.4.4 Transposition 72
2.4.5 The evolution of polyphony 74

References for Chapter 2 79

3 The Science of Sound 81
3.1 How We Hear 81
3.1.1 The magnificent ear-brain system 82
3.2 Attributes of Sound 84
3.2.1 Loudness and decibels 84
3.2.2 Frequency 86
3.3 Sine Waves 88
3.3.1 The sine function 89
3.3.2 Graphing sinusoids 91
3.3.3 The harmonic oscillator 94
3.4 Understanding Pitch 97
3.4.1 Residue pitch 98
3.4.2 A vibrating string 104
3.4.3 The overtone series 105
3.4.4 The starting transient 107
3.4.5 Resonance and beats 108
3.5 The Monochord Lab: Length versus Pitch 115

References for Chapter 3 118

4 Tuning and Temperament 119
4.1 The Pythagorean Scale 119
4.1.1 Consonance and integer ratios 120
4.1.2 The spiral of fifths 122
4.1.3 The overtone series revisited 124
4.2 Just Intonation 127
4.2.1 Problems with just intonation: The syntonic comma 129
4.2.2 Major versus minor 131
4.3 Equal Temperament 133
4.3.1 A conundrum and a compromise 133
4.3.2 Rational and irrational numbers 135
4.3.3 Cents 138
4.4 Comparing the Three Systems 141
4.5 Strähle’s Guitar 144
4.5.1 An ingenious construction 145
4.5.2 Continued fractions 149
4.5.3 On the accuracy of Strähle’s method 155
4.6 Alternative Tuning Systems 158
4.6.1 The significance of $\log_2(3/2)$ 158
4.6.2 Meantone scales 159
4.6.3 Other equally tempered scales 161

References for Chapter 4 163
## 5 Musical Group Theory

5.1 Symmetry in Music 165
5.1.1 Symmetric transformations 166
5.1.2 Inversions 169
5.1.3 Other examples 173
5.2 The Bartók Controversy 182
5.2.1 The Fibonacci numbers and nature 183
5.2.2 The golden ratio 184
5.2.3 *Music for Strings, Percussion and Celesta* 185
5.3 Group Theory 191
5.3.1 Some examples of groups 192
5.3.2 Multiplication tables 193
5.3.3 Symmetries of the square 195
5.3.4 The musical subgroup of $D_4$ 197

References for Chapter 5 201

## 6 Change Ringing 203

6.1 Basic Theory, Practice, and Examples 203
6.1.1 Nomenclature 204
6.1.2 Rules of an extent 205
6.1.3 Three bells 208
6.1.4 The number of permissible moves 210
6.1.5 Example: *Plain Bob Minimus* 211
6.1.6 Example: *Canterbury Minimus* 213
6.2 Group Theory Revisited 216
6.2.1 The symmetric group $S_n$ 216
6.2.2 The dihedral group revisited 218
6.2.3 Ringing the cosets 221
6.2.4 Example: *Plain Bob Doubles* 223

References for Chapter 6 227

## 7 Twelve-Tone Music 229

7.1 Schoenberg’s Twelve-Tone Method of Composition 229
7.1.1 Notation and terminology 230
7.1.2 The tone row matrix 233
7.2 Schoenberg’s *Suite für Klavier*, Op. 25 235
7.3 Tone Row Invariance 238
7.3.1 Using numbers instead of pitches 241
7.3.2 Further analysis: The symmetric interval property 242
7.3.3 Tritone symmetry 245
7.3.4 The number of distinct tone rows 248
7.3.5 Twelve-tone music and group theory 249

References for Chapter 7 252
8 Mathematical Modern Music 253
  8.1 Sir Peter Maxwell Davies: Magic Squares 253
    8.1.1 Magic squares 255
    8.1.2 Some examples 256
    8.1.3 The magic constant 258
    8.1.4 A Mirror of Whitening Light 260
  8.2 Steve Reich: Phase Shifting 268
    8.2.1 Clapping Music 271
    8.2.2 Phase shifts 276
  8.3 Xenakis: Stochastic Music 278
    8.3.1 A Greek architect 278
    8.3.2 Metastasis and the Philips Pavilion 279
    8.3.3 Pithoprakta: Continuity versus discontinuity 280
  8.4 Final Project: A Mathematical Composition 283
References for Chapter 8 287

Credits 289

Index 293